

NAME _____	DATE _____	PERIOD _____
3-1 Study Guide and Intervention Writing Equations		
Write Equations Writing equations is one strategy for solving problems. You can use a variable to represent an unspecified number or measure referred to in a problem. Then you can write a verbal expression as an algebraic expression.		
Example 1 Translate each sentence into an equation or a formula. a. Ten times a number x is equal to 2.8 times the difference y minus z. $10 \times x = 2.8 \times (y - z)$ The equation is $10x = 2.8(y - z)$. b. A number m minus 8 is the same as a number n divided by 2. $m - 8 = n \div 2$ The equation is $m - 8 = \frac{n}{2}$. c. The area of a rectangle equals the length times the width. Translate this sentence into a formula. Let $A =$ area, $\ell =$ length, and $w =$ width. Formula: <i>Area equals length times width.</i> $A = \ell \times w$ The formula for the area of a rectangle is $A = \ell w$.		
Example 2 Use the Four-Step Problem-Solving Plan. The population of the United States in 2001 was about 284,000,000, and the land area of the United States is about 3,500,000 square miles. Find the average number of people per square mile in the United States. Source: www.census.gov Step 1 Explore You know that there are 284,000,000 people. You want to know the number of people per square mile. Step 2 Plan Write an equation to represent the situation. Let p represent the number of people per square mile. $3,500,000 \times p = 284,000,000$ Step 3 Solve $3,500,000 \times p = 284,000,000$. Divide each side by 3,500,000. $p \approx 81.14$ There are about 81 people per square mile. Step 4 Examine If there are 81 people per square mile and there are 3,500,000 square miles, $81 \times 3,500,000 = 283,500,000$, or about 284,000,000 people. The answer makes sense.		
Example 3 Translate each sentence into an equation or formula. 1. Three times a number t minus twelve equals forty. $3t - 12 = 40$ 2. One-half of the difference of a and b is 54. $\frac{1}{2}(a - b) = 54$ 3. Three times the sum of d and 4 is 32. $3(d + 4) = 32$ 4. The area A of a circle is the product of π and the radius r squared. $A = \pi r^2$ WEIGHT LOSS For Exercises 5-6, use the following information. Lou wants to lose weight to audition for a part in a play. He weighs 160 pounds now. He wants to weigh 150 pounds. 5. If p represents the number of pounds he wants to lose, write an equation to represent this situation. $160 - p = 150$ 6. How many pounds does he need to lose to reach his goal? 10 lb		
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3-1 Study Guide and Intervention Writing Equations		
Write Verbal Sentences You can translate equations into verbal sentences.		
Example Translate each equation into a verbal sentence. a. $4n - 8 = 12$. $4n - 8 = 12$ Four times n minus eight equals twelve. b. $a^2 + b^2 = c^2$ $a^2 + b^2 = c^2$ The sum of the squares of a and b is equal to the square of c.		
Example 3 Translate each equation into a verbal sentence. 1. $4a - 5 = 23$ 2. $10 + k = 4k$ 4 times a minus 5 is equal to 23. The sum of 10 and k is equal to 4 times k. 3. $6xy = 24$ 4. $x^2 + y^2 = 8$ 6 times the product of x and y is equal to 24. The sum of the squares of x and y is equal to 8. 5. $p + 3 = 2p$ 6. $b = \frac{1}{3}(h - 1)$ The sum of p and 3 is equal to 2 times p. b is $\frac{1}{3}$ of the difference of h and 1. 7. $100 - 2x = 80$ 8. $3(g + h) = 12$ 100 minus 2 times x is equal to 80. 3 times the sum of g and h is 12. 9. $p^2 - 2p = 9$ 10. $C = \frac{5}{9}(F - 32)$ The square of p minus 2 times p is equal to 9. C is equal to $\frac{5}{9}$ of the difference of F and 32. 11. $V = \frac{1}{3}Bh$ 12. $A = \frac{1}{2}hb$ V is equal to $\frac{1}{3}$ of the product of B and h. A is equal to $\frac{1}{2}$ of the product of h and b.		
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3-1 Skills Practice

Writing Equations

Translate each sentence into an equation.

- Two added to three times a number m is the same as 18. **$3m + 2 = 18$**
- Twice a increased by the cube of a equals b . **$2a + a^3 = b$**
- Seven less than the sum of p and q is as much as 6. **$(p + q) - 7 = 6$**
- The sum of x and its square is equal to y times z . **$x + x^2 = yz$**
- Four times the sum of f and g is identical to six times g . **$4(f + g) = 6g$**

Translate each sentence into a formula.

- The perimeter P of a square equals four times the length of a side s . **$P = 4s$**
- The area A of a square is the length of a side s squared. **$A = s^2$**
- The perimeter P of a triangle is equal to the sum of the lengths of sides a , b , and c . **$P = a + b + c$**
- The area A of a circle is pi times the radius r squared. **$A = \pi r^2$**
- The volume V of a rectangular prism equals the product of the length ℓ , the width w , and the height h . **$V = \ell wh$**

Translate each equation into a verbal sentence.

- $g + 10 = 3g$
 g plus 10 is the same as 3 times g .
- $2p + 4q = 20$
Twice p plus 4 times q is 20.
- $4(a + b) = 9a$
4 times the sum of a and b is 9 times a .
- $\frac{1}{2}(f + y) = f - 5$
Half of the sum of f and y is f minus 5.
- $8 - 6x = 4 + 2x$
8 minus 6 times x is 4 plus 2 times x .
- $s^2 - n^2 = 2b$
 s squared minus n squared is twice b .

Write a problem based on the given information.

- $c =$ cost per pound of plain coffee beans
 $c + 3 =$ cost per pound of flavored coffee beans
 $2c + (c + 3) = 21$
Sample answer: The cost of two pounds of plain coffee beans plus one pound of flavored beans is \$21. How much does 1 pound of plain beans cost?
- $p =$ cost of dinner
 $0.15p =$ cost of a 15% tip
 $p + 0.15p = 23$
Sample answer: The cost of dinner plus a 15% tip was \$23. How much was the dinner?

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3-1 Practice (Average)

Writing Equations

Translate each sentence into an equation.

- Fifty-three plus four times c is as much as 21. **$53 + 4c = 21$**
- The sum of five times h and twice g is equal to 23. **$5h + 2g = 23$**
- One fourth the sum of r and ten is identical to r minus 4. **$\frac{1}{4}(r + 10) = r - 4$**
- Three plus the sum of the squares of w and x is 32. **$3 + (w^2 + x^2) = 32$**

Translate each sentence into a formula.

- Degrees Kelvin K equals 273 plus degrees Celsius C . **$K = 273 + C$**
- The total cost C of gas is the price p per gallon times the number of gallons g . **$C = pg$**
- The sum S of the measures of the angles of a polygon is equal to 180 times the difference of the number of sides n and 2. **$S = 180(n - 2)$**

Translate each equation into a verbal sentence.

- $q - (4 + p) = \frac{1}{3}q$
 q minus the sum of 4 and p equals $\frac{1}{3}$ times q .
- $\frac{3}{5}t + 2 = t$
Two more than $\frac{3}{5}$ of t equals t .
- $9(y^2 + x) = 18$
9 times the sum of y squared and x is 18.
- $2(m - n) = v + 7$
 m minus n is v plus 7.

Write a problem based on the given information.

- $a =$ cost of one adult's ticket to zoo
 $a - 4 =$ cost of one children's ticket to zoo
 $2a + 4(a - 4) = 38$
Sample answer: The cost of two adult's tickets and 4 children's tickets to the zoo is \$38. How much is an adult's ticket?
- $c =$ regular cost of one airline ticket
 $0.20c =$ amount of 20% promotional discount
 $3(c - 0.20c) = 330$
Sample answer: The cost of three airline tickets that are discounted 20% is \$330. What is the regular cost of a ticket?

- 14. GEOGRAPHY** About 15% of all federally-owned land in the 48 contiguous states of the United States is in Nevada. If F represents the area of federally-owned land in these states, and N represents the portion in Nevada, write an equation for this situation.
 $0.15F = N$

FITNESS For Exercises 15–17, use the following information.

- Deanna and Pietra each go for walks around a lake a few times per week. Last week, Deanna walked 7 miles more than Pietra.
- If p represents the number of miles Pietra walked, write an equation that represents the total number of miles T the two girls walked. **$T = p + (p + 7)$**
 - If Pietra walked 9 miles during the week, how many miles did Deanna walk? **16 mi**
 - If Pietra walked 11 miles during the week, how many miles did the two girls walk together? **29 mi**

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3-1 Reading to Learn Mathematics**Writing Equations****Pre-Activity** How are equations used to describe heights?

Read the introduction to Lesson 3-1 at the top of page 120 in your textbook.

Does the equation $305 - s = 154$ also represent the situation? Explain.**Yes; the total height minus the height of the statue itself gives the height of the pedestal.****Reading the Lesson**

1. Translate each sentence into an equation.

a.	Two times	the sum of x and three	minus	four	equals	four	times	x .
	2	$\times$		$(x + 3)$		4	$\times$	x

b.	The difference of k and 3	is	two times	k	divided by	five.
	$k - 3$	$=$	2	$\times$	k	$\div$ 5

2. A 1 oz serving of chips has 140 calories. There are about 14 servings of chips in a bag. How many calories are there in a bag of chips? Write what your solution would be as you use each step in the Four-Step Problem-Solving Plan.

Explore What do you know?**A 1 oz serving of chips has 140 calories and there are 14 servings of chips in a bag.**

What do you want to know?

How many calories are in a bag of chips?*Plan* Write an equation.

$140 \cdot 14 = x$

Solve Solve the problem.

$140 \cdot 14 = 1960$; There are 1960 calories in a bag of chips.

Examine Does your answer make sense?**See students' work.****Helping You Remember**

3. If you cannot remember all the steps of the Four-Step Problem-Solving Plan, try to remember the first letters of the first word in each step. Write those letters here with their associated words.

EPSE; Explore, Plan, Solve, Examine

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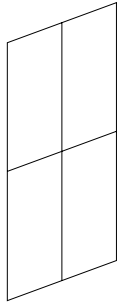
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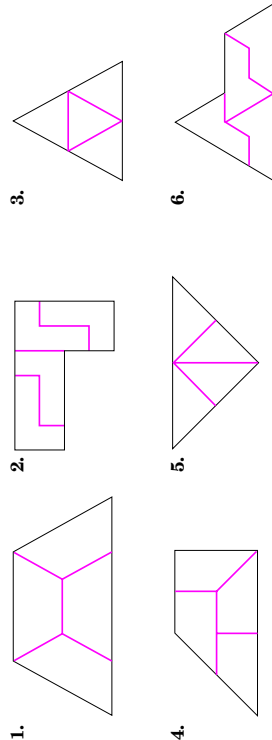
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3-1 Enrichment**Rep-Tiles**

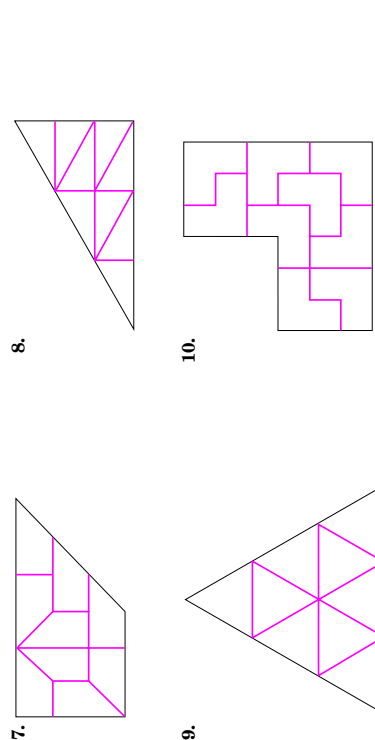
A rep-tile is a figure that can be subdivided into smaller copies of itself. The large figure is similar to the small ones and the small figures are all congruent.



Show that each figure is a rep-tile by subdividing it into four smaller and similar figures.



Subdivide each rep-tile into nine smaller and similar figures.



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3-2 Study Guide and Intervention

Solving Equations by Using Addition and Subtraction

Solve Using Addition If the same number is added to each side of an equation, the resulting equation is equivalent to the original one. In general if the original equation involves subtraction, this property will help you solve the equation.

Addition Property of Equality	For any numbers a , b , and c , if $a = b$, then $a + c = b + c$.
Example 1 Solve $m - 32 = 18$. $m - 32 = 18$ Original equation $m - 32 + 32 = 18 + 32$ Add 32 to each side. $m = 50$ Simplify. The solution is 50.	Example 2 Solve $-18 = p - 12$. $-18 = p - 12$ Original equation $-18 + 12 = p - 12 + 12$ Add 12 to each side. $p = -6$ Simplify. The solution is -6 .

Exercises

Solve each equation. Then check your solution.

1. $h - 3 = -2$ **1**
2. $m - 8 = -12$ **-4**
3. $p - 5 = 15$ **20**
4. $20 = y - 8$ **28**
5. $k - 0.5 = 2.3$ **2.8**
6. $w - \frac{1}{2} = \frac{5}{8}$ **$1\frac{1}{8}$**
7. $h - 18 = -17$ **1**
8. $-12 = -24 + k$ **12**
9. $j - 0.2 = 1.8$ **2**
10. $b - 40 = -40$ **0**
11. $m - (-12) = 10$ **-2**
12. $w - \frac{3}{2} = \frac{1}{4}$ **$\frac{7}{4}$**

Write an equation for each problem. Then solve the equation and check the solution.

13. Twelve subtracted from a number equals 25. Find the number. **$n - 12 = 25$; 37**
14. What number decreased by 52 equals -12 ? **$n - 52 = -12$; 40**
15. Fifty subtracted from a number equals eighty. Find the number. **$n - 50 = 80$; 130**
16. What number minus one-half is equal to negative one-half? **$n - \frac{1}{2} = -\frac{1}{2}$; 0**
17. The difference of a number and eight is equal to 14. What is the number?
 $n - 8 = 14$; 22
18. A number decreased by fourteen is equal to eighteen. What is the number?
 $n - 14 = 18$; 32

3-2 Study Guide and Intervention

Solving Equations by Using Addition and Subtraction

Solve Using Subtraction If the same number is subtracted from each side of an equation, the resulting equation is equivalent to the original one. In general if the original equation involves addition, this property will help you solve the equation.

Subtraction Property of Equality	For any numbers a , b , and c , if $a = b$, then $a - c = b - c$.
Example Solve $22 + p = -12$. $22 + p = -12$ Original equation $22 + p - 22 = -12 - 22$ Subtract 22 from each side. $p = -34$ Simplify. The solution is -34 .	

Exercises

Solve each equation. Then check your solution.

1. $x + 12 = 6$ **-6**
2. $z + 2 = -13$ **-15**
3. $-17 = b + 4$ **-21**
4. $s + (-9) = 7$ **16**
5. $-3.2 = \ell + (-0.2)$ **-3**
6. $-\frac{3}{8} + x = \frac{5}{8}$ **$1\frac{1}{4}$**
7. $19 + h = -4$ **-23**
8. $-12 = k + 24$ **-36**
9. $j + 1.2 = 2.8$ **1.6**
10. $b + 80 = -80$ **-160**
11. $m + (-8) = 2$ **10**
12. $w + \frac{3}{2} = \frac{5}{8}$ **$-\frac{7}{8}$**

Write an equation for each problem. Then solve the equation and check the solution.

13. Twelve added to a number equals 18. Find the number. **$n + 12 = 18$; 6**
14. What number increased by 20 equals -10 ? **$n + 20 = -10$; -30**
15. The sum of a number and fifty equals eighty. Find the number. **$n + 50 = 80$; 30**
16. What number plus one-half is equal to four? **$n + \frac{1}{2} = 4$; $3\frac{1}{2}$**
17. The sum of a number and 3 is equal to -15 . What is the number? **$n + 3 = -15$; -18**

Lesson 3-2

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Lesson 3-2

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3-2 Skills Practice

Solving Equations by Using Addition and Subtraction

Solve each equation. Then check your solution.

1. $y - 7 = 8$ **15**
2. $w + 14 = -8$ **-22**
3. $p - 4 = 6$ **10**
4. $-13 = 5 + x$ **-18**
5. $98 = b + 34$ **64**
6. $y - 32 = -1$ **31**
7. $s + (-28) = 0$ **28**
8. $y + (-10) = 6$ **16**
9. $-1 = s + (-19)$ **18**
10. $j - (-17) = 36$ **19**
11. $14 = d + (-10)$ **24**
12. $u + (-5) = -15$ **-10**
13. $11 = -16 + y$ **27**
14. $c - (-3) = 100$ **97**
15. $47 = w - (-8)$ **39**
16. $x - (-74) = -22$ **-96**
17. $4 - (-h) = 68$ **64**
18. $-56 = 20 - (-e)$ **-76**

Write an equation for each problem. Then solve the equation and check your solution.

19. A number decreased by 14 is -46. Find the number. **$n - 14 = -46$; -32**
20. Thirteen subtracted from a number is -5. Find the number. **$n - 13 = -5$; 8**
21. The sum of a number and 67 is equal to -34. Find the number. **$n + 67 = -34$; -101**
22. What number minus 28 equals -2? **$n - 28 = -2$; 26**
23. A number plus -73 is equal to 27. What is the number? **$n + (-73) = 27$; 100**
24. A number plus -17 equals -1. Find the number. **$n + (-17) = -1$; 16**
25. What number less 5 is equal to -39? **$n - 5 = -39$; -34**

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3-2 Practice (Average)

Solving Equations by Using Addition and Subtraction

Solve each equation. Then check your solution.

1. $d - 8 = 17$ **25**
2. $v + 12 = -5$ **-17**
3. $b - 2 = -11$ **-9**
4. $-16 = s + 71$ **-87**
5. $29 = a - 76$ **105**
6. $-14 + y = -2$ **12**
7. $8 - (-c) = 1$ **-7**
8. $78 + r = -15$ **-93**
9. $f + (-3) = -9$ **-6**
10. $4.2 = n + 7.3$ **-3.1**
11. $w + 1.9 = -2.5$ **-4.4**
12. $4.6 - (-b) = -0.4$ **-5**
13. $y - (-1.5) = 0.5$ **-1**
14. $a - 0.13 = -0.58$ **-0.45**
15. $k + (-4.21) = -19$ **-14.79**
16. $r + \frac{1}{5} = \frac{9}{10}$ **$\frac{7}{10}$**
17. $\frac{5}{9} + q = \frac{2}{3}$ **$\frac{1}{9}$**
18. $\frac{1}{3} = h + \frac{2}{5}$ **$-\frac{1}{15}$**
19. $\frac{1}{4} + x = -\frac{7}{12}$ **$-\frac{5}{6}$**
20. $y + \frac{4}{5} = \frac{3}{4}$ **$-\frac{1}{20}$**
21. $-\frac{7}{8} - (-n) = -\frac{7}{12}$ **$\frac{7}{24}$**

Write an equation for each problem. Then solve the equation and check your solution.

22. What number minus 9 is equal to -18? **$n - 9 = -18$; -9**
23. A number plus 15 equals -12. What is the number? **$n + 15 = -12$; -27**
24. The sum of a number and -3 is equal to -91. Find the number. **$n + (-3) = -91$; -88**
25. Negative seventeen equals 63 plus a number. What is the number? **$-17 = 63 + n$; -80**
26. The sum of negative 14, a number, and 6 is -5. What is the number? **$-14 + n + 6 = -5$; 3**
27. What number plus one half is equal to three eighths? **$n + \frac{1}{2} = \frac{3}{8}$; $-\frac{1}{8}$**

HISTORY For Exercises 28 and 29, use the following information.
Galileo Galilei was born in 1564. Many years later, in 1642, Sir Isaac Newton was born.

28. Write an addition equation to represent the situation. **$1564 + y = 1642$**
29. How many years after Galileo was born was Isaac Newton born? **78**

HURRICANES For Exercises 30 and 31, use the following information.
The day after a hurricane, the barometric pressure in a coastal town has risen to 29.7 inches of mercury, which is 2.9 inches of mercury higher than the pressure when the eye of the hurricane passed over.

30. Write an addition equation to represent the situation. **$b + 2.9 = 29.7$**
31. What was the barometric pressure when the eye passed over? **26.8 in. of mercury**

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3-2

Reading to Learn Mathematics

Solving Equations by Using Addition and Subtraction

Pre-Activity

How can equations be used to compare data?

Read the introduction to Lesson 3-2 at the top of page 128 in your textbook.

In the equation $m - 66 = 5$, the number 5 represents

the difference between the percent of growth for medical assistants and the percent of growth for travel agents,

and the number 66 represents

the rate of growth for travel agents.

Reading the Lesson

- To solve $x + 17 = 46$ using the Subtraction Property of Equality, you would subtract **17** from each side.
- To solve $y - 9 = -30$ using the Addition Property of Equality, you would add **9** to each side.
- Write an equation that you could solve by subtracting 32 from each side.
Sample answer: $m + 32 = 50$
- A student used the Subtraction Property of Equality to solve an equation. Explain why it would also be possible to use the Addition Property of Equality to solve the equation.
Subtracting one number from another gives the same result as adding the opposite of the number that was subtracted.

Helping You Remember

- Explain how you decide whether to use the Addition Property or the Subtraction Property of Equality to solve an equation.
Sample answer: If the given equation has a number added to the variable, then use the Subtraction Property of Equality. If the equation has a number subtracted from the variable, then use the Addition Property of Equality.

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3-2

Enrichment

Counting-Off Puzzles

Solve each puzzle.

1. Twenty-five people are standing in a circle. Starting with person 1, they count off from 1 to 7 and then start over with 1. Each person who says "7" drops out of the circle. Who is the last person left? **number 15**

2. Forty people stand in a circle. They count off so that every third person drops out. Which two people are the last ones left? **13th and 28th people**

3. Only half of the 30 students in Sharon's class can go on a field trip. Sharon arranges the boys and girls as shown. They count off from 1 to 9 and every ninth person drops out until only 15 people are left. Who gets to go on the field trip. **the girls**

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3-3 Study Guide and Intervention

Solving Equations by Using Multiplication and Division

Solve Using Multiplication If each side of an equation is multiplied by the same number, the resulting equation is equivalent to the given one. You can use the property to solve equations involving multiplication and division.

Multiplication Property of Equality For any numbers a , b , and c , if $a = b$, then $ac = bc$.

Example 1 Solve $3\frac{1}{2}p = 1\frac{1}{2}$.

Original equation
 $3\frac{1}{2}p = 1\frac{1}{2}$
Rewrite each mixed number as an improper fraction.
 $\frac{7}{2}p = \frac{3}{2}$
Multiply each side by $\frac{2}{7}$.
 $\frac{2}{7}(\frac{7}{2}p) = \frac{2}{7}(\frac{3}{2})$
Simplify.
 $p = \frac{3}{7}$
The solution is $\frac{3}{7}$.

Example 2 Solve $-\frac{1}{4}n = 16$.

Original equation
 $-\frac{1}{4}n = 16$
Multiply each side by -4 .
 $-4(-\frac{1}{4}n) = -4(16)$
Simplify.
 $n = -64$
The solution is -64 .

Example 1 Solve $8n = 64$.

Original equation
 $8n = 64$
Divide each side by 8.
 $\frac{8n}{8} = \frac{64}{8}$
Simplify.
 $n = 8$
The solution is 8.

Example 2 Solve $-5n = 60$.

Original equation
 $-5n = 60$
Divide each side by -5 .
 $\frac{-5n}{-5} = \frac{60}{-5}$
Simplify.
 $n = -12$
The solution is -12 .

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3-3 Study Guide and Intervention

Solving Equations by Using Multiplication and Division

Solve Using Division To solve equations with multiplication and division, you can also use the Division Property of Equality. If each side of an equation is divided by the same number, the resulting equation is true.

Division Property of Equality For any numbers a , b , and c , with $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

Example 1 Solve $8n = 64$.

Original equation
 $8n = 64$
Divide each side by 8.
 $\frac{8n}{8} = \frac{64}{8}$
Simplify.
 $n = 8$
The solution is 8.

Example 2 Solve $-5n = 60$.

Original equation
 $-5n = 60$
Divide each side by -5 .
 $\frac{-5n}{-5} = \frac{60}{-5}$
Simplify.
 $n = -12$
The solution is -12 .

Exercise 1

Solve each equation. Then check your solution.

- $3h = -42$ **-14**
- $8m = 16$ **2**
- $-3t = 51$ **-17**
- $-3r = -24$ **8**
- $8k = -64$ **-8**
- $-2m = 16$ **-8**
- $12h = 4$ **$\frac{1}{3}$**
- $-2.4p = 7.2$ **-3**
- $0.5j = 5$ **10**
- $-25 = 5m$ **-5**
- $6m = 15$ **$2\frac{1}{2}$**
- $-1.5p = -75$ **50**

Exercise 2

Write an equation for each problem. Then solve the equation.

- Four times a number equals 64. Find the number. **$4n = 64$; 16**
- What number multiplied by -4 equals -16 ? **$-4n = -16$; 4**
- A number times eight equals -36 . Find the number. **$8n = -36$; $-4\frac{1}{2}$**

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3-3 Study Guide and Intervention

Solving Equations by Using Multiplication and Division

Solve Using Multiplication If each side of an equation is multiplied by the same number, the resulting equation is equivalent to the given one. You can use the property to solve equations involving multiplication and division.

Multiplication Property of Equality For any numbers a , b , and c , if $a = b$, then $ac = bc$.

Example 1 Solve $3\frac{1}{2}p = 1\frac{1}{2}$.

Original equation
 $3\frac{1}{2}p = 1\frac{1}{2}$
Rewrite each mixed number as an improper fraction.
 $\frac{7}{2}p = \frac{3}{2}$
Multiply each side by $\frac{2}{7}$.
 $\frac{2}{7}(\frac{7}{2}p) = \frac{2}{7}(\frac{3}{2})$
Simplify.
 $p = \frac{3}{7}$
The solution is $\frac{3}{7}$.

Example 2 Solve $-\frac{1}{4}n = 16$.

Original equation
 $-\frac{1}{4}n = 16$
Multiply each side by -4 .
 $-4(-\frac{1}{4}n) = -4(16)$
Simplify.
 $n = -64$
The solution is -64 .

Exercise 1

Solve each equation. Then check your solution.

- $\frac{h}{3} = -2$ **-6**
- $\frac{1}{8}m = 6$ **48**
- $\frac{1}{5}p = \frac{3}{5}$ **3**
- $5 = \frac{y}{12}$ **60**
- $-\frac{1}{4}k = -2.5$ **10**
- $-\frac{m}{8} = \frac{5}{8}$ **-5**
- $-\frac{1}{2}h = 4$ **$-\frac{8}{3}$**
- $-12 = -\frac{3}{2}k$ **8**
- $\frac{j}{3} = \frac{2}{5}$ **$1\frac{1}{5}$**
- $-\frac{1}{3}b = 5$ **$-1\frac{1}{2}$**
- $\frac{7}{10}m = 10$ **$14\frac{2}{7}$**
- $\frac{p}{5} = -\frac{1}{4}$ **$-1\frac{1}{4}$**

Exercise 2

Write an equation for each problem. Then solve the equation.

- One-fifth of a number equals 25. Find the number. **$\frac{1}{5}n = 25$; 125**
- What number divided by 2 equals -18 ? **$\frac{n}{2} = -18$; -36**
- A number divided by eight equals 3. Find the number. **$\frac{n}{8} = 3$; 24**
- One and a half times a number equals 6. Find the number. **$1\frac{1}{2}n = 6$; 4**

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3-3 Skills Practice

Solving Equations by Using Multiplication and Division

Solve each equation. Then check your solution.

1. $12z = 108$ **9**
2. $-7t = 49$ **-7**
3. $18e = -216$ **-12**
4. $-22 = 11v$ **-2**
5. $-6d = -42$ **7**
6. $96 = -24a$ **-4**
7. $\frac{c}{4} = 16$ **64**
8. $\frac{a}{16} = 9$ **144**
9. $-84 = \frac{d}{3}$ **-252**
10. $-\frac{d}{7} = -13$ **91**
11. $\frac{f}{4} = -13$ **-52**
12. $31 = -\frac{1}{6}n$ **-186**
13. $-6 = \frac{2}{3}z$ **-9**
14. $\frac{2}{7}q = -4$ **-14**
15. $\frac{5}{9}p = -10$ **-18**
16. $\frac{a}{10} = \frac{2}{5}$ **4**
17. $-0.4b = 5.2$ **-13**
18. $1.6m = -4$ **-2.5**

Write an equation for each problem. Then solve the equation.

19. The opposite of a number is -9 . What is the number? **$-n = -9$; 9**
20. Fourteen times a number is -42 . Find the number. **$14n = -42$; -3**
21. Eight times a number equals 128. What is the number? **$8n = 128$; 16**
22. Negative twelve times a number equals -132 . Find the number. **$-12n = -132$; 11**
23. Negative eighteen times a number is -54 . What is the number? **$-18n = -54$; 3**
24. One sixth of a number is -17 . Find the number. **$\frac{1}{6}n = -17$; -102**
25. Negative three fifths of a number is -15 . What is the number? **$-\frac{3}{5}n = -15$; 25**

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3-3 Practice (Average)

Solving Equations by Using Multiplication and Division

Solve each equation. Then check your solution.

1. $8j = 96$ **12**
2. $-13z = -39$ **3**
3. $-180 = 15m$ **-12**
4. $243 = 27c$ **9**
5. $\frac{y}{9} = -8$ **-72**
6. $-\frac{j}{12} = -8$ **96**
7. $\frac{a}{15} = \frac{4}{5}$ **12**
8. $\frac{g}{27} = \frac{2}{9}$ **6**
9. $\frac{q}{24} = \frac{1}{6}$ **4**
10. $-1 = -\frac{4}{7}t$ **$\frac{7}{4}$**
11. $-\frac{3}{8}w = -9$ **24**
12. $-\frac{3}{15}s = 4$ **-20**
13. $-3x = \frac{3}{2}$ **$-\frac{1}{2}$**
14. $\frac{8}{5}a = \frac{4}{3}$ **$\frac{5}{6}$**
15. $\frac{5}{3}h = \frac{11}{6}$ **$\frac{11}{10}$**
16. $5n = \frac{11}{4}$ **$\frac{11}{20}$**
17. $2.5k = 20$ **8**
18. $-3.4e = -3.74$ **1.1**
19. $-1.7b = 2.21$ **-1.3**
20. $0.26p = 0.104$ **0.4**
21. $4.2q = -3.36$ **-0.8**

Write an equation for each problem. Then solve the equation.

22. Negative nine times a number equals -117 . Find the number. **$-9n = -117$; 13**
23. Negative one eighth of a number is $-\frac{3}{4}$. What is the number? **$-\frac{1}{8}n = -\frac{3}{4}$; 6**
24. Five sixths of a number is $-\frac{5}{9}$. Find the number. **$\frac{5}{6}n = -\frac{5}{9}$; $-\frac{2}{3}$**
25. 2.7 times a number equals 8.37. What is the number? **$2.7n = 8.37$; 3.1**
26. One and one fourth times a number is one and one third. What is the number?
 $1\frac{1}{4}n = 1\frac{1}{3}$; $1\frac{1}{15}$

27. PUBLISHING Two units of measure used in publishing are the *pica* and the *point*. A pica is one sixth of an inch. There are 12 points in a pica, so Points = 12 · Picas. How many picas are equivalent to 108 points? **9 picas**

ROLLER COASTERS For Exercises 28 and 29, use the following information.

Superman the Escape in California is the fastest roller coaster in the world. Riders fall 415 feet in 7 seconds. Speeds reach a maximum of 100 miles per hour.

28. If x represents the average rate of fall of the roller coaster, write an expression to represent the situation (*Hint*: Use the distance formula $d = rt$.) **$7x = 415$**

29. What is the average rate that riders fall in feet per second? **about 59.3 ft/s**

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3-3 Reading to Learn Mathematics

Solving Equations by Using Multiplication and Division

Pre-Activity How can equations be used to find how long it takes light to reach Earth?

Read the introduction to Lesson 3-3 at the top of page 135 in your textbook.

- In the equation $d = rt$, shown in the introduction, what number is used for r ? for d ?

5,870,000,000,000; 311,110,000,000,000

- What equation could you use to find the time it takes light to reach Earth from the farthest star in the Big Dipper?

5,870,000,000,000t = 821,800,000,000

Reading the Lesson

Complete the sentence after each equation to tell how you would solve the equation.

1. $\frac{x}{7} = 16$ **Multiply** each side by **7**.
2. $5x = 125$ **Divide** each side by **5**, or multiply each side by **$\frac{1}{5}$** .
3. $-8k = 96$ Divide each side by **-8**, or multiply each side by **$-\frac{1}{8}$** .
4. Explain how rewriting $4\frac{1}{3}x = 2\frac{1}{8}$ as $\frac{13}{3}x = \frac{17}{8}$ helps you solve the equation.

It makes it easier for you to see what number you need to multiply by on each side.

Helping You Remember

5. One way to remember something is to explain it to someone else. Write how you would explain to a classmate how to solve the equation $\frac{2}{3}x = 12$.

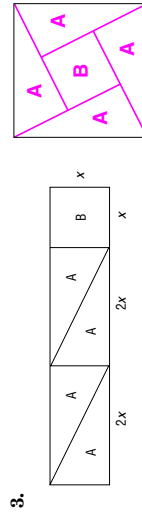
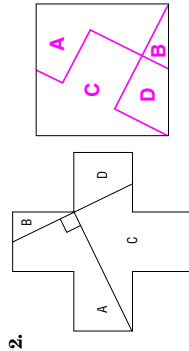
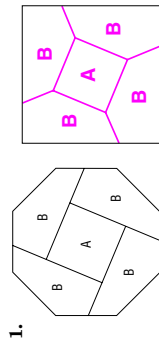
Sample answer: Multiply each side of the equation by the reciprocal of $\frac{2}{3}$ so you can isolate x on the left side.

3-3 Enrichment

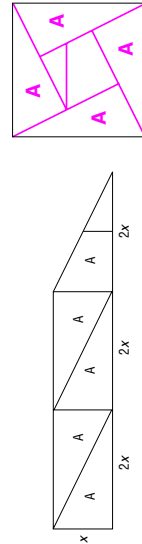
Dissection Puzzles: Make the Square

In a dissection puzzle, you are to cut apart one figure using only straight cuts and then rearrange the pieces to make a new figure. Usually the puzzle-solver must figure out where to make the given number of cuts. However, for these puzzles, the cut lines are shown. You must discover how to rearrange the pieces.

Cut apart each figure. Then rearrange the pieces to form a square.



4. *Hint:* Cut one of the triangles into two pieces to make this square.



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3-4 Study Guide and Intervention

Solving Multi-Step Equations

Work Backward Working backward is one of many problem-solving strategies that you can use to solve problems. To work backward, start with the result given at the end of a problem and undo each step to arrive at the beginning number.

Example 1 A number is divided by 2, and then 8 is subtracted from the quotient. The result is 16. What is the number?

Solve the problem by working backward. The final number is 16. Undo subtracting 8 by adding 8 to get 24. To undo dividing 24 by 2, multiply 24 by 2 to get 48. The original number is 48.

Example 2 A bacteria culture doubles each half hour. After 3 hours, there are 6400 bacteria. How many bacteria were there to begin with?

Solve the problem by working backward. The bacteria have grown for 3 hours. Since there are 2 one-half hour periods in one hour, in 3 hours there are 6 one-half hour periods. Since the bacteria culture has grown for 6 time periods, it has doubled 6 times. Undo the doubling by halving the number of bacteria 6 times.

$$6,400 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 6,400 \times \frac{1}{64} = 100$$

There were 100 bacteria to begin with.

Exercises

Solve each problem by working backward.

- A number is divided by 3, and then 4 is added to the quotient. The result is 8. Find the number. **12**
- A number is multiplied by 5, and then 3 is subtracted from the product. The result is 12. Find the number. **3**
- Eight is subtracted from a number, and then the difference is multiplied by 2. The result is 24. Find the number. **20**
- Three times a number plus 3 is 24. Find the number. **7**
- CAR RENTAL** Angela rented a car for \$29.99 a day plus a one-time insurance cost of \$5.00. Her bill was \$124.96. For how many days did she rent the car? **4 days**
- MONEY** Mike withdrew an amount of money from his bank account. He spent one fourth for gasoline and had \$90 left. How much money did he withdraw? **\$120**
- TELEVISIONS** In 1999, 68% of households with TV's subscribed to cable TV. If 8,000 more subscribers are added to the number of households with cable, the total number of households with cable TV would be 67,600,000. How many households were there with TV in 1999? **Source: World Almanac 99,400,000 households**

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3-4 Study Guide and Intervention

Solving Multi-Step Equations

Solve Multi-Step Equations To solve equations with more than one operation, often called **multi-step equations**, undo operations by working backward. Reverse the usual order of operations as you work.

Example Solve $5x + 3 = 23$.

$5x + 3 = 23$ Original equation.
 $5x + 3 - 3 = 23 - 3$ Subtract 3 from each side.
 $5x = 20$ Simplify.
 $\frac{5x}{5} = \frac{20}{5}$ Divide each side by 5.
 $x = 4$ Simplify.

Exercises

Solve each equation. Then check your solution.

- $5x + 2 = 27$ **5**
- $2.6x + 9 = 27$ **3**
- $5x + 16 = 51$ **7**
- $14n - 8 = 34$ **3**
- $0.6x - 1.5 = 1.8$ **5.5**
- $\frac{7}{8}p - 4 = 10$ **16**
- $16 = \frac{d - 12}{14}$ **236**
- $8.8 + \frac{3n}{12} = 13$ **20**
- $\frac{g}{-5} + 3 = -13$ **80**
- $\frac{4b + 8}{-2} = 10$ **-7**
- $0.2x - 8 = -2$ **30**
- $3.2y - 1.8 = 3$ **1.5**
- $-4 = \frac{7x - (-1)}{-8}$ **$4\frac{3}{7}$**
- $8 = -12 + \frac{k}{-4}$ **-80**
- $0 = 10y - 40$ **4**

Write an equation and solve each problem.

- Find three consecutive integers whose sum is 96.
 $n + (n + 1) + (n + 2) = 96$; 31, 32, 33
- Find two consecutive odd integers whose sum is 176.
 $n + (n + 2) = 176$; 87, 89
- Find three consecutive integers whose sum is -93.
 $n + (n + 1) + (n + 2) = -93$; -32, -31, -30

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3-4 Skills Practice Solving Multi-Step Equations Solve each problem by working backward. 1. A number is divided by 2, and then the quotient is added to 8. The result is 33. Find the number. 50 2. Two is subtracted from a number, and then the difference is divided by 3. The result is 30. Find the number. 92 3. A number is multiplied by 2, and then the product is added to 9. The result is 49. What is the number? 20 4. ALLOWANCE After Ricardo received his allowance for the week, he went to the mall with some friends. He spent half of his allowance on a new paperback book. Then he bought himself a snack for \$1.25. When he arrived home, he had \$5.00 left. How much was his allowance? \$12.50 Solve each equation. Then check your solution. 5. $5x + 3 = 23$ 4 6. $4 = 3a - 14$ 6 7. $2y + 5 = 19$ 7 8. $6 + 5c = -29$ -7 9. $8 - 5w = -37$ 9 10. $18 - 4v = 42$ -6 11. $\frac{n}{3} - 8 = -2$ 18 12. $5 + \frac{x}{4} = 1$ -16 13. $-\frac{h}{3} - 4 = 13$ -51 14. $-\frac{d}{6} + 12 = -7$ 114 15. $\frac{a}{5} - 2 = 9$ 55 16. $\frac{w}{7} + 3 = -1$ -28 17. $\frac{3}{4}q - 7 = 8$ 20 18. $\frac{2}{3}g + 6 = -12$ -27 19. $\frac{5}{2}z - 8 = -3$ 2 20. $\frac{4}{5}m + 2 = 6$ 5 21. $\frac{c - 5}{4} = 3$ 17 22. $\frac{b + 1}{3} = 2$ 5 Write an equation and solve each problem. 23. Twice a number plus four equals 6. What is the number? $2n + 4 = 6$; 1 24. Sixteen is seven plus three times a number. Find the number. $16 = 7 + 3n$; 3 25. Find two consecutive integers whose sum is 35. $n + (n + 1) = 35$; 17, 18 26. Find three consecutive integers whose sum is 36. $n + (n + 1) + (n + 2) = 36$; 11, 12, 13			Lesson 3-4	
NAME _____ DATE _____ PERIOD _____				
3-4 Practice (Average) Solving Multi-Step Equations Solve each problem by working backward. 1. Three is added to a number, and then the sum is multiplied by 4. The result is 16. Find the number. 1 2. A number is divided by 4, and the quotient is added to 3. The result is 24. What is the number? 84 3. Two is subtracted from a number, and then the difference is multiplied by 5. The result is 30. Find the number. 8 4. BIRD WATCHING While Michelle sat observing birds at a bird feeder, one fourth of the birds flew away when they were startled by a noise. Two birds left the feeder to go to another stationed a few feet away. Three more birds flew into the branches of a nearby tree. Four birds remained at the feeder. How many birds were at the feeder initially? 12 Solve each equation. Then check your solution. 5. $-12n - 19 = 77$ -8 6. $17 + 3f = 14$ -1 7. $15t + 4 = 49$ 3 8. $\frac{u}{5} + 6 = 2$ -20 9. $\frac{d}{-4} + 3 = 15$ -48 10. $\frac{b}{3} - 6 = -2$ 12 11. $\frac{1}{2}y - \frac{1}{8} = \frac{7}{8}$ 2 12. $-32 - \frac{3}{5}f = -17$ -25 13. $8 - \frac{3}{8}k = -4$ 32 14. $\frac{r + 13}{12} = 1$ -1 15. $\frac{15 - a}{3} = -9$ 42 16. $\frac{3k - 7}{5} = 16$ 29 17. $\frac{x}{7} - 0.5 = 2.5$ 21 18. $2.5g + 0.45 = 0.95$ 0.2 19. $0.4m - 0.7 = 0.22$ 2.3 Write an equation and solve each problem. 20. Seven less than four times a number equals 13. What is the number? $4n - 7 = 13$; 5 21. Find two consecutive odd integers whose sum is 116. $n + (n + 2) = 116$; 57, 59 22. Find two consecutive even integers whose sum is 126. $n + (n + 2) = 126$; 62, 64 23. Find three consecutive odd integers whose sum is 117. $n + (n + 2) + (n + 4) = 117$; 37, 39, 41 24. COIN COLLECTING Jung has a total of 92 coins in his coin collection. This is 8 more than three times the number of quarters in the collection. How many quarters does Jung have in his collection? 28			© Glencoe/McGraw-Hill 157 Glencoe Algebra 1	
NAME _____ DATE _____ PERIOD _____				
3-4 Practice (Average) Solving Multi-Step Equations Solve each problem by working backward. 1. Three is added to a number, and then the sum is multiplied by 4. The result is 16. Find the number. 1 2. A number is divided by 4, and the quotient is added to 3. The result is 24. What is the number? 84 3. Two is subtracted from a number, and then the difference is multiplied by 5. The result is 30. Find the number. 8 4. BIRD WATCHING While Michelle sat observing birds at a bird feeder, one fourth of the birds flew away when they were startled by a noise. Two birds left the feeder to go to another stationed a few feet away. Three more birds flew into the branches of a nearby tree. Four birds remained at the feeder. How many birds were at the feeder initially? 12 Solve each equation. Then check your solution. 5. $-12n - 19 = 77$ -8 6. $17 + 3f = 14$ -1 7. $15t + 4 = 49$ 3 8. $\frac{u}{5} + 6 = 2$ -20 9. $\frac{d}{-4} + 3 = 15$ -48 10. $\frac{b}{3} - 6 = -2$ 12 11. $\frac{1}{2}y - \frac{1}{8} = \frac{7}{8}$ 2 12. $-32 - \frac{3}{5}f = -17$ -25 13. $8 - \frac{3}{8}k = -4$ 32 14. $\frac{r + 13}{12} = 1$ -1 15. $\frac{15 - a}{3} = -9$ 42 16. $\frac{3k - 7}{5} = 16$ 29 17. $\frac{x}{7} - 0.5 = 2.5$ 21 18. $2.5g + 0.45 = 0.95$ 0.2 19. $0.4m - 0.7 = 0.22$ 2.3 Write an equation and solve each problem. 20. Seven less than four times a number equals 13. What is the number? $4n - 7 = 13$; 5 21. Find two consecutive odd integers whose sum is 116. $n + (n + 2) = 116$; 57, 59 22. Find two consecutive even integers whose sum is 126. $n + (n + 2) = 126$; 62, 64 23. Find three consecutive odd integers whose sum is 117. $n + (n + 2) + (n + 4) = 117$; 37, 39, 41 24. COIN COLLECTING Jung has a total of 92 coins in his coin collection. This is 8 more than three times the number of quarters in the collection. How many quarters does Jung have in his collection? 28			© Glencoe/McGraw-Hill 158 Glencoe Algebra 1	

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3-4 Reading to Learn Mathematics

Solving Multi-Step Equations

Pre-Activity How can equations be used to estimate the age of an animal?

Read the introduction to Lesson 3-4 at the top of page 142 in your textbook.

- Write the equation $8 + 12a = 124$ in words.
Eight plus twelve times a equals one hundred twenty-four.
- How many operations are involved in the equation? **two**

Reading the Lesson

1. What does the phrase *undo the operations* mean to you? Give an example.
Using the opposite operations in the opposite order undoes the operations; subtraction undoes addition.
2. a. If we undo operations in reverse of the order of operations, what operations do we do first? **addition or subtraction**
b. What operations do we do last? **multiplication or division**
3. Suppose you want to solve $\frac{x + 3}{5} = 6$.
a. What is the grouping symbol in the equation $\frac{x + 3}{5} = 6$? **the fraction bar**
b. What is the first step in solving the equation? **Multiply each side by 5.**
c. What is the next step in solving the equation? **Subtract 3 from each side.**
4. Write an equation for the problem below.

Seven	times	k	minus	five	equals	negative forty-seven
7	•	k	-	5	=	-47

Helping You Remember

5. Explain why working backward is a useful strategy for solving equations.
Sample answer: You can undo the operations to get back to the value of the variable that will make the equation true. That value is the solution.

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3-4 Enrichment

Consecutive Integer Problems

Many types of problems and puzzles involve the idea of consecutive integers. Knowing how to represent these integers algebraically can help to solve the problem.

Example Find four consecutive odd integers whose sum is -80.

An odd integer can be written as $2n + 1$, where n is any integer. If $2n + 1$ is the first odd integer, then add 2 to get the next largest odd integer, and so on.

Now write an equation to solve this problem.

$$(2n + 1) + (2n + 3) + (2n + 5) + (2n + 7) = -80$$

Exercises

Write an equation for each problem. Then solve.

1. Complete the solution to the problem in the example.
-23, -21, -19, -17
2. Find three consecutive even integers whose sum is 132.
 $2n + (2n + 2) + (2n + 4) = 132$; $n = 21$; 42, 44, 46
3. Find two consecutive integers whose sum is 19.
 $n + (n + 1) = 19$; 9, 10
4. Find two consecutive integers whose sum is 100.
 $n + (n + 1) = 100$; no solution
5. The lesser of two consecutive even integers is 10 more than one-half the greater. Find the integers.
 $2n = 10 + \frac{1}{2}(2n + 2)$; 22 and 24
6. The greater of two consecutive even integers is 6 less than three times the lesser. Find the integers.
 $2n + 2 = 3(2n) - 6$; 4, 6
7. Find four consecutive integers such that twice the sum of the two greater integers exceeds three times the first by 91.
 $2[(n + 2) + (n + 3)] = 3n + 91$; 81, 82, 83, 84
8. Find a set of four consecutive positive integers such that the greatest integer in the set is twice the least integer in the set.
 $n + 3 = 2n$; {3, 4, 5, 6}

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3-5 Skills Practice**Solving Equations with the Variable on Each Side**

Justify each step.

1. $4k - 3 = 2k + 5$
 $4k - 3 - 2k = 2k + 5 - 2k$
 $2k - 3 = 5$
 $2k - 3 + 3 = 5 + 3$
 $2k = 8$
 $\frac{2k}{2} = \frac{8}{2}$
 $k = 4$
 - a. Subtract $2k$ from each side.
 - b. Simplify.
 - c. Add 3 to each side.
 - d. Simplify.
 - e. Divide each side by 2.
 - f. Simplify.
2. $2(8u + 2) = 3(2u - 7)$
 $16u + 4 = 6u - 21$
 $16u + 4 - 6u = 6u - 21 - 6u$
 $10u + 4 = -21$
 $10u + 4 - 4 = -21 - 4$
 $10u = -25$
 $\frac{10u}{10} = \frac{-25}{10}$
 $u = -2.5$
 - a. Distributive Property
 - b. Subtract $6u$ from each side.
 - c. Simplify.
 - d. Subtract 4 from each side.
 - e. Simplify.
 - f. Divide each side by 10.
 - g. Simplify.

Solve each equation. Then check your solution.

3. $2m + 12 = 3m - 31$ **43**
5. $7a - 3 = 3 - 2a$ **$2\frac{2}{3}$**
7. $4x - 9 = 7x + 12$ **-7**
9. $5 + 3r = 5r - 19$ **12**
11. $8q + 12 = 4(3 + 2q)$ **all numbers**
13. $6(-3v + 1) = 5(-2v - 2)$ **2**
15. $3(8 - 3t) = 5(2 + t)$ **1**
17. $8(2f - 2) = 7(3f + 2)$ **-6**
19. $6(w - 1) = 3(3w + 5)$ **-7**
21. $\frac{2}{3}v - 6 = 6 - \frac{2}{3}v$ **9**
4. $2h - 8 = h + 17$ **25**
6. $4n - 12 = 12 - 4n$ **3**
8. $-6y - 3 = 3 - 6y$ **no solution**
10. $-9 + 8k = 7 + 4k$ **4**
12. $3(5j + 2) = 2(3j - 6)$ **-2**
14. $-7(2b - 4) = 5(-2b + 6)$ **-0.5 or $-\frac{1}{2}$**
16. $2(3u + 7) = -4(3 - 2u)$ **13**
18. $5(-6 - 3d) = 3(8 + 7d)$ **-1.5 or $-1\frac{1}{2}$**
20. $7(-3y + 2) = 8(3y - 2)$ **$\frac{2}{3}$**
22. $\frac{1}{2} - \frac{5}{8}x = \frac{7}{8}x + \frac{7}{2}$ **-2**

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3-5 Practice (Average)**Solving Equations with the Variable on Each Side**

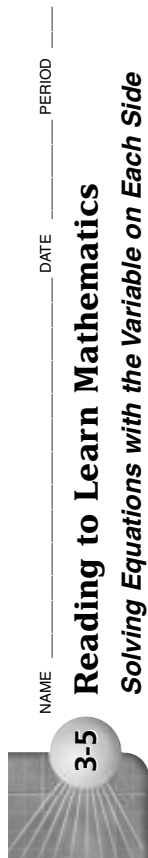
Solve each equation. Then check your solution.

1. $5x - 3 = 13 - 3x$ **2**
3. $1 - s = 6 - 6s$ **1**
5. $\frac{1}{2}k - 3 = 2 - \frac{3}{4}k$ **4**
7. $3(-2 - 3x) = -9x - 4$ **no solution**
9. $9(4b - 1) = 2(9b + 3)$ **$\frac{5}{6}$**
11. $-5x - 10 = 2 - (x + 4)$ **-2**
13. $\frac{5}{2}t - t = 3 + \frac{3}{2}$ **no solution**
15. $\frac{2}{3}x - \frac{1}{6} = \frac{1}{2}x + \frac{5}{6}$ **6**
17. $\frac{1}{2}(3g - 2) = \frac{g}{6}$ **$\frac{3}{4}$**
19. $\frac{1}{4}(5 - 2k) = \frac{h}{2}$ **$1\frac{1}{4}$**
21. $3(d - 8) - 5 = 9(d + 2) + 1$ **-8**
23. Two thirds of a number reduced by 11 is equal to 4 more than the number. Find the number. **-45**
25. Five times the sum of a number and 3 is the same as 3 multiplied by 1 less than twice the number. What is the number? **18**
27. Tripling the greater of two consecutive even integers gives the same result as subtracting 10 from the lesser even integer. What are the integers? **-8, -6**
29. The formula for the perimeter of a rectangle is $P = 2\ell + 2w$, where ℓ is the length and w is the width. A rectangle has a perimeter of 24 inches. Find its dimensions if its length is 3 inches greater than its width. **4.5 in. by 7.5 in.**
2. $-4c - 11 = 4c + 21$ **-4**
4. $14 + 5n = -4n + 17$ **$\frac{1}{3}$**
6. $\frac{1}{2}(6 - z) = z$ **2**
8. $4(4 - w) = 3(2w + 2)$ **1**
10. $3(6 + 5y) = 2(-5 + 4y)$ **-4**
12. $6 + 2(3j - 2) = 4(1 + j)$ **1**
14. $1.4t + 1.1 = 8.3 - f$ **3**
16. $2 - \frac{3}{4}z = \frac{1}{8}z + 9$ **-8**
18. $\frac{1}{3}(c + 1) = \frac{1}{6}(3c - 5)$ **7**
20. $\frac{1}{9}(2m - 16) = \frac{1}{3}(2m + 4)$ **-7**
22. $2(a - 8) + 7 = 5(a + 2) - 3a - 19$ **all numbers**

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3-5 Reading to Learn Mathematics

Solving Equations with the Variable on Each Side

Pre-Activity How can an equation be used to determine when two populations are equal?

Read the introduction to Lesson 3-5 at the top of page 149 in your textbook.

In the equation $12 + 7.6x = 6 + 8x$, what do $7.6x$ and $8x$ represent?

$7.6x$ represents the increase (in millions) in the number of male Internet users, and $8x$ represents the increase (in millions) in the number of female Internet users.

Reading the Lesson

1. Suppose you want to help a friend solve $6k + 7 = 3k - 8$. What would you advise her to do first? Why?

Subtract $3k$ from each side; after she does that and simplifies, all of the variables will be on the left side.

2. When solving $2(3x - 4) = 3(x + 5)$, why is it helpful first to use the Distributive Property to remove the grouping symbols?

Once you have removed the grouping symbols, you can tell what you need to add or subtract to each side to get all of the variables on one side.

3. On a quiz, Jason solved three equations. His teacher said all the work was correct, but she asked him to write short sentences to tell what the solutions were. In what follows, you see the *last* equation in his work for each equation. Write sentences to describe the solutions.

- a. $x = -4$ **The solution is -4 .**
- b. $6m = 6m$ **All numbers are solutions.**
- c. $12 = 37$ **There are no solutions.**

4. In Question 3, one of the equations Jason solved was an identity. Which equation was it? Explain how you know.

The one for which the last step was $6m = 6m$; the expressions on the left and right sides are the same.

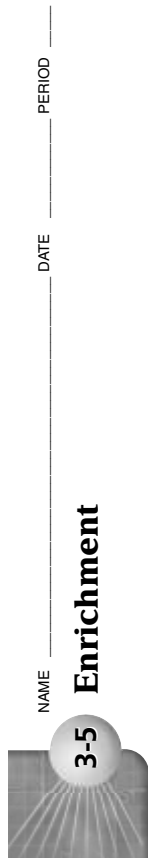
Helping You Remember

5. An equation with variables is an identity when the equation is always true. In other words, the expressions on the left and right sides always have the same value. Look up the word *identity* in the dictionary. Write all the definitions that are similar to the mathematical definition. **See students' work.**

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3-5 Enrichment

Identities

An equation that is true for every value of the variable is called an **identity**. When you try to solve an identity, you end up with a statement that is always true. Here is an example.

Example Solve $8 - (5 - 6x) = 3(1 + 2x)$.

$$8 - (5 - 6x) = 3(1 + 2x)$$

$$8 - 5 - (-6x) = 3(1 + 2x)$$

$$8 - 5 + 6x = 3 + 6x$$

$$3 + 6x = 3 + 6x$$

Exercises

State whether each equation is an identity. If it is not, find its solution.

1. $2(2 - 3x) = 3(3 + x) + 4$

$x = -1$

identity

3. $(5t + 9) - (3t - 13) = 2(11 + t)$

identity

4. $14 - (6 - 3c) = 4c - c$

no solution

5. $3y - 2(y + 19) = 9y - 3(9 - y)$

$y = -1$

6. $3(3h - 1) = 4(h + 3)$

$h = 3$

7. Use the true equation $3x - 2 = 3x - 2$ to create an identity of your own.

Sample answer: $6x - 4 = 2(3x - 2)$

8. Use the false equation $1 = 2$ to create an equation with no solution.

Sample answer: $2x + 1 = 2x + 2$

9. Create an equation whose solution is $x = 3$.

Sample answer: $3x + 2(x + 1) = 20$

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3-6

Study Guide and Intervention

Ratios and Proportions

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Ratios and Proportions A ratio is a comparison of two numbers by division. The ratio of x to y can be expressed as x to y , xy or $\frac{x}{y}$. Ratios are usually expressed in simplest form. An equation stating that two ratios are equal is called a **proportion**. To determine whether two ratios form a proportion, express both ratios in simplest form or check cross products.

Example 1 Determine whether the ratios $\frac{24}{36}$ and $\frac{12}{18}$ form a proportion.

$\frac{24}{36} = \frac{2}{3}$ when expressed in simplest form.
 $\frac{12}{18} = \frac{2}{3}$ when expressed in simplest form.
The ratios $\frac{24}{36}$ and $\frac{12}{18}$ form a proportion because they are equal when expressed in simplest form.

Example 2 Use cross products to determine whether $\frac{10}{18}$ and $\frac{25}{45}$ form a proportion.

$\frac{10}{18} \cdot \frac{25}{45}$ Write the proportion.
 $10(45) \neq 18(25)$ Cross products
 $450 \neq 450$ Simplify.
The cross products are equal, so $\frac{10}{18} = \frac{25}{45}$.
Since the ratios are equal, they form a proportion.

Exercises

Use cross products to determine whether each pair of ratios forms a proportion.

1. $\frac{1}{2}, \frac{32}{16}$
yes

2. $\frac{5}{8}, \frac{15}{16}$
no

3. $\frac{12}{32}, \frac{3}{16}$
no

4. $\frac{25}{36}, \frac{15}{20}$
no

5. $\frac{15}{20}, \frac{9}{12}$
yes

6. $\frac{0.1}{2}, \frac{5}{100}$
yes

7. $\frac{2}{3}, \frac{20}{30}$
yes

8. $\frac{1.5}{2}, \frac{6}{8}$
yes

9. $\frac{5}{9}, \frac{27}{9}$
yes

10. $\frac{14}{21}, \frac{20}{30}$
yes

11. 5 to 9, 25 to 45
yes

12. $\frac{72}{64}, \frac{9}{8}$
yes

13. 5:5, 30:20
no

14. 18 to 24, 50 to 75
no

15. 100:75, 44:33
yes

16. $\frac{0.05}{1}, \frac{5}{20}$
yes

17. $\frac{1.5}{2}, \frac{0.45}{0.9}$
yes

3-6

Study Guide and Intervention

Ratios and Proportions

NAME _____ DATE _____ PERIOD _____

Solve Proportions If a proportion involves a variable, you can use cross products to solve the proportion. In the proportion $\frac{x}{5} = \frac{10}{13}$, x and 13 are called **extremes** and 5 and 10 are called **means**. In a proportion, the product of the extremes is equal to the product of the means.

Means-Extremes Property of Proportions

For any numbers a , b , c , and d , if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Example Solve $\frac{x}{5} = \frac{10}{13}$.

$\frac{x}{5} = \frac{10}{13}$ Original proportion
 $13(x) = 5(10)$ Cross products
 $13x = 50$ Simplify.
 $\frac{13x}{13} = \frac{50}{13}$ Divide each side by 13.
 $x = 3\frac{11}{13}$ Simplify.
The solution is $3\frac{11}{13}$.

Exercises

Solve each proportion.

1. $\frac{-3}{x} = \frac{2}{8}$
-12

2. $\frac{1}{t} = \frac{5}{3}$
10

3. $\frac{0.1}{2} = \frac{0.5}{x}$
10

4. $\frac{x+1}{4} = \frac{3}{4}$
2

5. $\frac{4}{6} = \frac{8}{x}$
12

6. $\frac{x}{21} = \frac{3}{63}$
1

7. $\frac{9}{y+1} = \frac{18}{54}$
26

8. $\frac{3}{d} = \frac{18}{3}$
1

9. $\frac{5}{8} = \frac{p}{24}$
15

10. $\frac{4}{b-2} = \frac{4}{12}$
14

11. $\frac{1.5}{x} = \frac{12}{x}$
no solution

12. $\frac{3+y}{4} = \frac{-y}{8}$
-2

13. $\frac{a-8}{12} = \frac{15}{3}$
68

14. $\frac{12}{k} = \frac{24}{k}$
no solution

15. $\frac{2+w}{6} = \frac{12}{9}$
6

Use a proportion to solve each problem.

- 16. MODELS** To make a model of the Guadeloupe River bed, Hermie used 1 inch of clay for 5 miles of the river's actual length. His model river was 50 inches long. How long is the Guadeloupe River? **250 mi**
- 17. EDUCATION** Josh finished 24 math problems in one hour. At that rate, how many hours will it take him to complete 72 problems? **3 h**

NAME _____	DATE _____	PERIOD _____
3-6 Skills Practice		
Ratios and Proportions		
Use cross products to determine whether each pair of ratios forms a proportion. Write <i>yes</i> or <i>no</i> .		
1. $\frac{4}{5}, \frac{20}{25}$	2. $\frac{5}{9}, \frac{7}{11}$	3. $\frac{6}{7}, \frac{24}{28}$
4. $\frac{20}{25}, \frac{4}{5}$	5. $\frac{7}{11}, \frac{5}{9}$	6. $\frac{28}{24}, \frac{7}{6}$
7. $\frac{24}{28}, \frac{6}{7}$	8. $\frac{26}{38}, \frac{13}{19}$	9. $\frac{38}{26}, \frac{19}{13}$
10. $\frac{38}{26}, \frac{19}{13}$	11. $\frac{50}{85}, \frac{12}{17}$	12. $\frac{85}{50}, \frac{17}{12}$
13. $\frac{42}{30}, \frac{7}{5}$	14. $\frac{26}{38}, \frac{13}{19}$	15. $\frac{38}{26}, \frac{19}{13}$
16. $\frac{30}{42}, \frac{5}{7}$	17. $\frac{50}{85}, \frac{12}{17}$	18. $\frac{85}{50}, \frac{17}{12}$
19. $\frac{42}{30}, \frac{7}{5}$	20. $\frac{26}{38}, \frac{13}{19}$	21. $\frac{38}{26}, \frac{19}{13}$
22. $\frac{30}{42}, \frac{5}{7}$	23. $\frac{50}{85}, \frac{12}{17}$	24. $\frac{85}{50}, \frac{17}{12}$
25. $\frac{42}{30}, \frac{7}{5}$	26. $\frac{26}{38}, \frac{13}{19}$	27. $\frac{38}{26}, \frac{19}{13}$
28. $\frac{30}{42}, \frac{5}{7}$	29. $\frac{50}{85}, \frac{12}{17}$	30. $\frac{85}{50}, \frac{17}{12}$
31. $\frac{42}{30}, \frac{7}{5}$	32. $\frac{50}{85}, \frac{12}{17}$	33. $\frac{85}{50}, \frac{17}{12}$
34. PAINTING Ysidra paints a room that has 400 square feet of wall space in $2\frac{1}{2}$ hours. At this rate, how long will it take her to paint a room that has 720 square feet of wall space? $4\frac{1}{2}$ h	35. VACATION PLANS Walker is planning a summer vacation. He wants to visit Petrified National Forest and Meteor Crater, Arizona, the 50,000-year-old impact site of a large meteor. On a map with a scale where 2 inches equals 75 miles, the two areas are about $1\frac{1}{2}$ inches apart. What is the distance between Petrified National Forest and Meteor Crater? about 56.25 mi	
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3-6 Reading to Learn Mathematics**Ratios and Proportions****Pre-Activity** How are ratios used in recipes?

Read the introduction to Lesson 3-6 at the top of page 155 in your textbook.

- How many servings of honey frozen yogurt are made by this recipe?
4 servings
- How many recipes would be needed to make enough honey frozen yogurt for all the students in your class? **See students' work.**

Reading the Lesson

- Complete the following sentence.
A ratio is a comparison of two numbers by **division**.
- Describe two ways to decide whether the sentence $\frac{2}{5} = \frac{8}{20}$ is a proportion.
Express the ratios in simplest form to see if they are equal. Check to see whether the cross products are equal.
- For each proportion, tell what the extremes are and what the means are.
 a. $\frac{14}{35} = \frac{6}{15}$ Extremes: **14 and 15** Means: **35 and 6**
 b. $\frac{6}{8} = \frac{12}{16}$ Extremes: **6 and 16** Means: **8 and 12**
- A jet flying at a steady speed traveled 825 miles in 2 hours. If you solved the proportion $\frac{825}{2} = \frac{x}{1.5}$, what would the answer tell you about the jet?
how far the jet traveled in 1.5 h

Helping You Remember

- Write how you would explain solving a proportion to a friend who missed Lesson 3-6.
Use cross products. Write an equation with the product of the extremes on the left side and the product of the means on the right side. Then solve this second equation.

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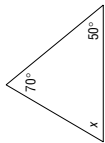
3-6 Enrichment**Angles of a Triangle**

In geometry, many statements about physical space are proven to be true. Such statements are called **theorems**. Here are two examples of geometric theorems.

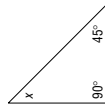
- The sum of the measures of the angles of a triangle is 180° .
- If two sides of a triangle have equal measure, then the two angles opposite those sides also have equal measure.

For each of the triangles, write an equation and then solve for x . (A tick mark on two or more sides of a triangle indicates that the sides have equal measure.)

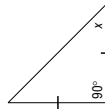
- 1.
- $x = 60^\circ$



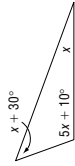
- 2.
- $x = 45^\circ$



- 3.
- $x = 45^\circ$



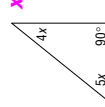
- 4.
- $x = 20^\circ$



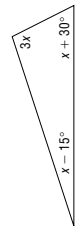
- 5.
- $x = 22.5^\circ$



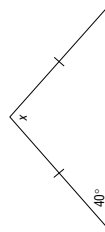
- 6.
- $x = 10^\circ$



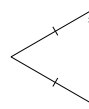
- 7.
- $x = 33^\circ$



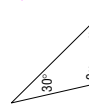
- 8.
- $x = 100^\circ$



- 9.
- $x = 60^\circ$



- 10.
- $x = 50^\circ$



- Two angles of a triangle have the same measure. The sum of the measures of these angles is one-half the measure of the third angle. Find the measures of the angles of the triangle.
 $30^\circ, 30^\circ, 120^\circ$

- The measure of one angle of a triangle is twice the measure of a second angle. The measure of the third angle is 12 less than the sum of the other two. Find the measures of the angles of the triangle.
 $64^\circ, 32^\circ, 84^\circ$

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3-7 Study Guide and Intervention Percent of Change Percent of Change When an increase or decrease in an amount is expressed as a percent, the percent is called the percent of change. If the new number is greater than the original number, the percent of change is a percent of increase. If the new number is less than the original number, the percent of change is the percent of decrease. Example 1 Find the percent of increase. original: 48 new: 60 First, subtract to find the amount of increase. The amount of increase is $60 - 48 = 12$. Then find the percent of increase by using the original number, 48, as the base. $\frac{12}{48} = \frac{r}{100}$ Percent proportion $12(100) = 48(r)$ Cross products $1200 = 48r$ Simplify. $\frac{1200}{48} = \frac{48r}{48}$ Divide each side by 48. $25 = r$ The percent of increase is 25%. Example 2 Find the percent of decrease. original: 30 new: 22 First, subtract to find the amount of decrease. The amount of decrease is $30 - 22 = 8$. Then find the percent of decrease by using the original number, 30, as the base. $\frac{8}{30} = \frac{r}{100}$ Percent proportion $8(100) = 30(r)$ Cross products $800 = 30r$ Simplify. $\frac{800}{30} = \frac{30r}{30}$ Divide each side by 30. $26\frac{2}{3} = r$ Simplify. The percent of decrease is $26\frac{2}{3}\%$, or about 27%. Example 3 State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent. - original: 50 new: 80 increase; 60% - original: 90 new: 100 increase; 11% - original: 45 new: 20 decrease; 56% - original: 77.5 new: 62 decrease; 20% - original: 140 new: 150 increase; 7% - original: 135 new: 90 decrease; 33% - original: 120 new: 180 increase; 50% - original: 90 new: 270 increase; 200% - original: 84 new: 98 increase; 17% - original: 12.5 new: 10 decrease; 20% - original: 250 new: 500 increase; 100% © Glencoe/McGraw-Hill	3-7 Study Guide and Intervention Percent of Change Solve Problems Discounted prices and prices including tax are applications of percent of change. Discount is the amount by which the regular price of an item is reduced. Thus, the discounted price is an example of percent of decrease. Sales tax is amount that is added to the cost of an item, so the price including tax is an example of percent of increase. Example A coat is on sale for 25% off the original price. If the original price of the coat is \$75, what is the discounted price? The discount is 25% of the original price. $25\% \text{ of } \$75 = 0.25 \times 75 \quad 25\% = 0.25$ Use a calculator. $= 18.75$ Subtract \$18.75 from the original price. $\$75 - \$18.75 = \$56.25$ The discounted price of the coat is \$56.25. Example 3 Find the final price of each item. When a discount and a sales tax are listed, compute the discount price before computing the tax. - Compact disc: \$16 Discount: 15% \$13.60 - Two concert tickets: \$28 Student discount: 28% \$20.16 - Airline ticket: \$248.00 Superair discount: 33% \$166.16 - Shirt: \$24.00 Sales tax: 4% \$24.96 - CD player: \$142.00 Sales tax: 5.5% \$149.81 - Celebrity calendar: \$10.95 Sales tax: 7.5% \$11.77 - Class ring: \$89.00 Group discount: 17% Sales tax: 5% \$77.56 - Software: \$44.00 Discount: 21% Sales tax: 6% \$36.85 - Video recorder: \$110.95 Discount: 20% Sales tax: 5% \$93.20 VIDEOS The original selling price of a new sports video was \$65.00. Due to the demand the price was increased to \$87.75. What was the percent of increase over the original price? 35% SCHOOL A high school paper increased its sales by 75% when it ran an issue featuring a contest to win a class party. Before the contest issue, 10% of the school's 800 students bought the paper. How many students bought the contest issue? 140 students BASEBALL Baseball tickets cost \$15 for general admission or \$20 for box seats. The sales tax on each ticket is 8%, and the municipal tax on each ticket is an additional 10% of the base price. What is the final cost of each type of ticket? \$17.70; \$23.60 © Glencoe/McGraw-Hill



3-7 Skills Practice

Percent of Change

NAME _____ DATE _____ PERIOD _____

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

- original: 25
new: 10
decrease; 60%
- original: 55
new: 50
decrease; 9%
- original: 50
new: 30
decrease; 40%
- original: 48
new: 60
increase; 25%
- original: 25
new: 75
increase; 50%
- original: 25
new: 28
increase; 12%
- original: 90
new: 95
increase; 6%
- original: 60
new: 45
decrease; 25%
- binder: \$14.50
tax: 7%
\$15.52
- groceries: \$47.52
tax: 3%
\$48.95
- shoes: \$65.00
tax: 4%
\$67.60
- concert tickets: \$48.00
tax: 7.5%
\$51.60
- monitor: \$150.00
discount: 50%
\$75.00
- shirt: \$25.50
discount: 40%
\$15.30
- coffee maker: \$102.00
discount: 45%
\$56.10



3-7 Skills Practice

Percent of Change

NAME _____ DATE _____ PERIOD _____

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

- original: 25
new: 10
decrease; 60%
- original: 55
new: 50
decrease; 9%
- original: 50
new: 30
decrease; 40%
- original: 48
new: 60
increase; 25%
- original: 25
new: 75
increase; 50%
- original: 25
new: 28
increase; 12%
- original: 90
new: 95
increase; 6%
- original: 60
new: 45
decrease; 25%
- binder: \$14.50
tax: 7%
\$15.52
- groceries: \$47.52
tax: 3%
\$48.95
- shoes: \$65.00
tax: 4%
\$67.60
- concert tickets: \$48.00
tax: 7.5%
\$51.60
- monitor: \$150.00
discount: 50%
\$75.00
- shirt: \$25.50
discount: 40%
\$15.30
- coffee maker: \$102.00
discount: 45%
\$56.10



3-7 Practice (Average)

Percent of Change

NAME _____ DATE _____ PERIOD _____

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

- original: 18
new: 10
decrease; 44%
- original: 10
new: 25
increase; 150%
- original: 15
new: 35.5
increase; 137%
- original: 76
new: 60
decrease; 21%
- original: 98.6
new: 64
decrease; 35%
- original: 140
new: 160
increase; 14%
- original: 128
new: 120
decrease; 6%
- original: 58.8
new: 65.7
increase; 12%
- concrete blocks: \$95.00
tax: 6%
\$100.70
- crib: \$240.00
tax: 6.5%
\$255.60
- blanket: \$24.99
tax: 7%
\$26.74
- class ring: \$325.00
tax: 6%
\$344.50
- jacket: \$125.00
tax: 5.5%
\$131.88
- kite: \$18.90
tax: 5%
\$19.85
- computer game: \$49.99
discount: 25%
\$37.49
- luggage: \$185.00
discount: 30%
\$129.50
- stationery: \$12.95
discount: 10%
\$11.66
- prescription glasses: \$149
discount: 20%
\$119.20
- pair of shorts: \$24.99
discount: 45%
\$13.74
- DVD player: \$269.00
discount: 20%
tax: 7%
\$230.26
- printer: \$255.00
discount: 30%
tax: 5.5%
\$188.32

25. INVESTMENTS The price per share of an internet-related stock decreased from \$90 per share to \$36 per share early in 2001. By what percent did the price of the stock decrease? **60%**

26. HEATING COSTS Customers of a utility company received notices in their monthly bills that heating costs for the average customer had increased 125% over last year because of an unusually severe winter. In January of last year, the Garcia's paid \$120 for heating. What should they expect to pay this January if their bill increased by 125%? **\$270**



3-7 Practice (Average)

Percent of Change

NAME _____ DATE _____ PERIOD _____

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

- original: 18
new: 10
decrease; 44%
- original: 10
new: 25
increase; 150%
- original: 15
new: 35.5
increase; 137%
- original: 76
new: 60
decrease; 21%
- original: 98.6
new: 64
decrease; 35%
- original: 140
new: 160
increase; 14%
- original: 128
new: 120
decrease; 6%
- original: 58.8
new: 65.7
increase; 12%
- concrete blocks: \$95.00
tax: 6%
\$100.70
- crib: \$240.00
tax: 6.5%
\$255.60
- blanket: \$24.99
tax: 7%
\$26.74
- class ring: \$325.00
tax: 6%
\$344.50
- jacket: \$125.00
tax: 5.5%
\$131.88
- kite: \$18.90
tax: 5%
\$19.85
- computer game: \$49.99
discount: 25%
\$37.49
- luggage: \$185.00
discount: 30%
\$129.50
- stationery: \$12.95
discount: 10%
\$11.66
- prescription glasses: \$149
discount: 20%
\$119.20
- pair of shorts: \$24.99
discount: 45%
\$13.74
- DVD player: \$269.00
discount: 20%
tax: 7%
\$230.26
- printer: \$255.00
discount: 30%
tax: 5.5%
\$188.32

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3-7

Reading to Learn Mathematics

Percent of Change

Pre-Activity How can percents describe growth over time?

Read the introduction to Lesson 3-7 at the top of page 160 in your textbook.

- How many area codes were in use in 1947? **84 area codes**
- How many *more* area codes were in use in 1999? **201 area codes**

Reading the Lesson

- If you use (original amount) — (new amount) to find the change for a percent of change problem, then the problem involves a percent of **decrease** (increase/decrease).
- If you use (new amount) — (original amount) to find the change for a percent of change problem, then the problem involves a percent of **increase** (increase/decrease).

Complete the chart.

Original Amount	New Amount	Percent Proportion	Percent Increase or Percent Decrease?
3. 10	13	$\frac{\text{change}}{\text{original}} \rightarrow \frac{3}{10} = \frac{r}{100}$	increase
4. 10	7	$\frac{\text{change}}{\text{original}} \rightarrow \frac{3}{10} = \frac{r}{100}$	decrease
5. 50	42	$\frac{\text{change}}{\text{original}} \rightarrow \frac{8}{50} = \frac{r}{100}$	decrease
6. 50	58	$\frac{\text{change}}{\text{original}} \rightarrow \frac{8}{50} = \frac{r}{100}$	increase

- When you find a discount price, do you add to or subtract from the original price?
subtract

Helping You Remember

- If you remember only two things about the ratio used for finding percent of change, what should they be? **Subtract the prices, then divide by the original number.**

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3-7

Enrichment

Using Percent

Use what you have learned about percent to solve each problem.

A TV movie had a "rating" of 15 and a 25 "share." The rating is the percentage of the nation's total TV households that were tuned in to this show. The share is the percentage of homes with TVs turned on that were tuned to the movie. How many TV households had their TVs turned off at this time?

To find out, let T = the number of TV households
and x = the number of TV households with the TV off.

Then $T - x$ = the number of TV households with the TV on.

Since 0.15 T and 0.25($T - x$) both represent the number of households tuned to the movie,

$$\begin{aligned} 0.15T &= 0.25(T - x) \\ 0.15T &= 0.25T - 0.25x \\ \text{Solve for } x: \quad 0.25x &= 0.10T \\ x &= \frac{0.10T}{0.25} = 0.40T \end{aligned}$$

Forty percent of the TV households had their TVs off when the movie was aired.

Answer each question.

- During that same week, a sports broadcast had a rating of 22.1 and a 43 share. Show that the percent of TV households with their TVs off was about 48.6%.
 $0.221T = 0.43T - 0.43x$
 $x = \frac{0.221T - 0.43T}{0.25} = 0.486T$
- Find the percent of TV households with their TVs turned off during a show with a rating of 18.9 and a 29 share. **34.8%**
- Show that if T is the number of TV households, r is the rating, and s is the share, then the number of TV households with the TV off is $\frac{s-r}{s}$.
Solve $rT = s(T - x)$ for x .
- If the fraction of TV households with no TV on is $\frac{s-r}{s}$, then show that the fraction of TV households with TVs on is $\frac{r}{s} \cdot 1 - \frac{s-r}{s} = \frac{r}{s}$.
- Find the percent of TV households with TVs on during the most watched serial program in history: the last episode of *M*A*S*H*, which had a 60.3 rating and a 77 share. **$\frac{60.3}{77} = 78.3\%$**
- A local station now has a 2 share. Each share is worth \$50,000 in advertising revenue per month. The station is thinking of going commercial free for the three months of summer to gain more listeners. What would its new share have to be for the last 4 months of the year to make more money for the year than it would have made had it not gone commercial free? **greater than 3.5**

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3-8 Study Guide and Intervention

Solving Equations and Formulas

Solve for Variables Sometimes you may want to solve an equation such as $V = \ell wh$ for one of its variables. For example, if you know the values of V , w , and h , then the equation $\ell = \frac{V}{wh}$ is more useful for finding the value of ℓ . If an equation that contains more than one variable is to be solved for a specific variable, use the properties of equality to isolate the specified variable on one side of the equation.

Example 1

Solve $2x - 4y = 8$ for y .

$$\begin{aligned} 2x - 4y &= 8 \\ 2x - 4y - 2x &= 8 - 2x \\ -4y &= 8 - 2x \\ \frac{-4y}{-4} &= \frac{8 - 2x}{-4} \\ y &= \frac{8 - 2x}{-4} \text{ or } \frac{2x - 8}{4} \end{aligned}$$

The value of y is $\frac{2x - 8}{4}$.

Example 2

Solve $3m - n = km - 8$ for m .

$$\begin{aligned} 3m - n &= km - 8 \\ 3m - n - km &= km - 8 - km \\ 3m - n - km &= -8 \\ 3m - n - km + n &= -8 + n \\ 3m - km &= -8 + n \\ m(3 - k) &= -8 + n \\ \frac{m(3 - k)}{3 - k} &= \frac{-8 + n}{3 - k} \\ m &= \frac{-8 + n}{3 - k}, \text{ or } \frac{n - 8}{3 - k} \end{aligned}$$

The value of m is $\frac{n - 8}{3 - k}$. Since division by 0 is undefined, $3 - k \neq 0$, or $k \neq 3$.

Exercises

Solve each equation or formula for the variable specified.

- $ax - b = c$ for x
 $x = \frac{c + b}{a}, a \neq 0$
- $15x + 1 = y$ for x
 $x = \frac{y - 1}{15}$
- $x(4 - k) = p$ for k
 $k = 4 - \frac{p}{x}, x \neq 0$
- $xy + z = 9$ for y
 $y = \frac{9 - z}{x}, x \neq 0$
- $2x + b = c$ for x
 $x = \frac{c - b}{2}$
- $4(c + 3) = t$ for c
 $c = \frac{t}{4} - 3$
- $16z + 4x = y$ for x
 $x = \frac{y - 16z}{4}$
- $d = rt$ for r
 $r = \frac{d}{t}, t \neq 0$
- $2\ell + 2w = w$
 $w = \frac{P - 2\ell}{2}$
- $C = \frac{5}{9}(F - 32)$ for F
 $F = \frac{9}{5}C + 32$
- $(x + f) + 2 = j$ for x
 $x = j - 2 - f$
- $7x + 3y = m$ for y
 $y = \frac{m - 7x}{3}$
- $x(1 + y) = z$ for x
 $x = \frac{z}{1 + y}, y \neq -1$
- $A = \frac{h(a + b)}{2}$ for h
 $h = \frac{2A}{a + b}, a \neq -b$
- $A = \ell w$ for ℓ
 $\ell = \frac{A}{w}, w \neq 0$

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3-8 Study Guide and Intervention

Solving Equations and Formulas

Use Formulas Many real-world problems require the use of formulas. Sometimes solving a formula for a specified variable will help solve the problem.

Example

The formula $C = \pi d$ represents the circumference of a circle, or the distance around the circle, where d is the diameter. If an airplane could fly around Earth at the equator without stopping, it would have traveled about 24,900 miles. Find the diameter of Earth.

$$C = \pi d \quad \text{Given formula}$$

$$d = \frac{C}{\pi} \quad \text{Solve for } d$$

$$d = \frac{24,900}{3.14} \quad \text{Use } \pi \approx 3.14$$

$$d \approx 7930 \quad \text{Simplify.}$$

The diameter of Earth is about 7930 miles.

Exercises

- GEOMETRY** The volume of a cylinder V is given by the formula $V = \pi r^2 h$, where r is the radius and h is the height.
 - Solve the formula for h . $h = \frac{V}{\pi r^2}$
 - Find the height of a cylinder with volume 2500π feet and radius 10 feet. **25 ft**
- WATER PRESSURE** The water pressure on a submerged object is given by $P = 64d$, where P is the pressure in pounds per square foot, and d is the depth of the object in feet.
 - Solve the formula for d . $d = \frac{P}{64}$
 - Find the depth of a submerged object if the pressure is 672 pounds per square foot. **10.5 ft**
- GRAPHS** The equation of a line containing the points $(a, 0)$ and $(0, b)$ is given by the formula $\frac{x}{a} + \frac{y}{b} = 1$.
 - Solve the equation for y . $y = b\left(1 - \frac{x}{a}\right)$
 - Suppose the line contains the points $(4, 0)$, and $(0, -2)$. If $x = 3$, find y . $-\frac{1}{2}$
- GEOMETRY** The surface area of a rectangular solid is given by the formula $S = 2\ell w + 2\ell h + 2wh$, where ℓ = length, w = width, and h = height.
 - Solve the formula for h . $h = \frac{S - 2\ell w}{2\ell + 2w}$
 - The surface area of a rectangular solid with length 6 centimeters and width 3 centimeters is 72 square centimeters. Find the height. **2 cm**

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3-8 Skills Practice Solving Equations and Formulas Solve each equation or formula for the variable specified. $7t = x$, for t $t = \frac{x}{7}$ $q - r = r$, for r $r = \frac{q}{2}$ $7a - b = 15a$, for a $a = -\frac{b}{8}$ $x - 2y = 1$, for y $y = \frac{x-1}{2}$ $7f + g = 5$, for f $f = \frac{5-g}{7}$ $rt - 2n = y$, for t $t = \frac{2n+y}{r}$ $kn + 4f = 9v$, for n $n = \frac{9v-4f}{k}$ $\frac{x-c}{2} = d$, for x $x = c + 2d$ $\frac{p+9}{5} = q$, for p $p = 5q - 9$ - Five more than a number g is six less than twice a number h. Solve for g. $g + 5 = 2h - 6$; $g = 2h - 11$ - One fourth of a number q is three more than three times a number w. Solve for q. $\frac{1}{4}q = 3w + 3$; $q = 12w + 12$ - Eight less than a number s is three more than four times a number t. Solve for s. $s - 8 = 4t + 3$; $s = 4t + 11$ © Glencoe/McGraw-Hill	3-8 Practice (Average) Solving Equations and Formulas Solve each equation or formula for the variable specified. $d = rt$, for r $r = \frac{d}{t}$ $mx + 4y = 3c$, for x $x = \frac{3c-4y}{m}$ $ab + 3c = 2d$, for b $b = \frac{2d-3c}{a}$ $\frac{2}{3}m + a = a + c$, for m $m = \frac{3c}{2}$ $\frac{2}{3}y + v = s$, for y $y = \frac{3(s-v)}{2}$ $\frac{rx+9}{5} = h$, for x $x = \frac{5h-9}{r}$ $2w - y = 7w - 2$, for w $w = \frac{2-y}{5}$ - Three times a number s plus 4 times a number y is 1 more than 6 times the number s. Solve for s. $3s + 4y = 6s + 1$; $s = \frac{4y-1}{3}$ - Five times a number k minus 9 is two thirds of a number j. Solve for j. $5k - 9 = \frac{2}{3}j$; $j = \frac{3}{2}(5k - 9)$ ELECTRICITY For Exercises 17 and 18, use the following information. The formula for Ohm's Law is $E = IR$, where E represents voltage measured in volts, I represents current measured in amperes, and R represents resistance measured in ohms. - Solve the formula for R. $R = \frac{E}{I}$ - Suppose a current of 0.25 ampere flows through a resistor connected to a 12-volt battery. What is the resistance in the circuit? 48 ohms MOTION For Exercises 19 and 20, use the following information. In <i>uniform circular motion</i>, the speed v of a point on the edge of a spinning disk is $v = \frac{2\pi}{T}r$, where r is the radius of the disk and T is the time it takes the point to travel once around the circle. - Solve the formula for r. $r = \frac{Tv}{2\pi}$ - Suppose a merry-go-round is spinning once every 3 seconds. If a point on the outside edge has a speed of 12.56 feet per second, what is the radius of the merry-go-round? (Use 3.14 for π.) 6 ft © Glencoe/McGraw-Hill
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3-8 Reading to Learn Mathematics

Solving Equations and Formulas

Pre-Activity How are equations used to design roller coasters?

Read the introduction to Lesson 3-8 at the top of page 166 in your textbook.

The equation $g(195 - h) = \frac{1}{2}v^2$ contains several variables. What number values do you know for these variables in this situation?
32 for g and 49 for v

Reading the Lesson

1. Suppose you have an equation with several variables. You want to solve for a particular variable. How does the procedure compare with that for solving an equation with just one variable? How does the solution compare with the solution for an equation with one variable?

Sample answer: The procedure is basically the same. You use properties of operations and equality to isolate the variable in which you are interested. The solution will probably contain variables instead of just a number.

2. Describe what dimensional analysis involves.

Sample answer: You use the units along with number values as you do calculations. You treat the units pretty much the same way you would variables. For example, you can divide them and use exponents with them.

3. What do you have to be careful about when you use variables in denominators of fractions?

You have to be sure that you do not use values that would make the denominator 0.

Helping You Remember

4. When you give the dimensions of a rectangle, you have to tell how many units long it is and how many units wide it is. How can this help you remember what dimensional analysis involves.

Sample answer: Keep the words *dimension* and *unit* in mind. In dimensional analysis, you include units for dimensions in calculations.

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3-8 Enrichment

Dr. Bernardo Houssay

Even though researchers have been studying the disease *diabetes mellitus* for hundreds of years, scientists have only recently discovered the cause of the disease and developed methods for reducing its severity. Dr. Bernardo Houssay, an Argentine physiologist, was one of the pioneers of this more modern research. He studied the relationship between diabetes and the pituitary gland, and in 1947 became the first Latin American to win the Nobel Prize in Medicine and Physiology.

Though there is no cure for diabetes, specific diets and exercise can help people control the disease. The American Diabetes Association (ADA) has helped establish flexible dietary guidelines for consumers to follow. These guidelines include some of the following general nutrition rules.

- Fat intake should be equal to or less than 30% of daily calories.
- Saturated fat intake should be equal to or less than 10% of daily calories.
- Protein should be limited to 10% to 20% of daily calories. Persons showing the initial signs of diabetes-induced kidney disease should limit protein to 10% of daily calories.
- Cholesterol intake should be 300 milligrams or less daily.

Refer to the information above for Exercises 1–4.

1. Robert consumed 2100 calories on Tuesday. His fat intake totaled 70 grams, and of that 70 grams, 14 were saturated.

a. What percentage of his calorie consumption was fat, and what percentage of that fat was saturated? (To find the percentage of calories from fat, multiply the number of fat grams by 9 before dividing by the number of calories.) **30%; 20%**

b. Did Robert stay within the recommended allowance of fats? **yes**

2. Anna's cholesterol intake was 330 milligrams on Sunday. By what percentage does she need to reduce her cholesterol consumption to remain within the guidelines? **10%**

3. What number of fat grams is 30% of 240 calories? **8**

4. Sharon follows a diet that provides about 50 grams of protein each day. Sharon's doctor has just told her to reduce her daily protein intake by 30%. About how much protein will be in her reduced protein diet? **35 grams**

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3-9 Study Guide and Intervention (continued)

Weighted Averages

Uniform Motion Problems Motion problems are another application of weighted averages. **Uniform motion problems** are problems where an object moves at a certain speed, or rate. Use the formula $d = rt$ to solve these problems, where d is the distance, r is the rate, and t is the time.

Example Bill Gutierrez drove at a speed of 65 miles per hour on an expressway for 2 hours. He then drove for 1.5 hours at a speed of 45 miles per hour on a state highway. What was his average speed?

$$M = \frac{65 \cdot 2 + 45 \cdot 1.5}{2 + 1.5}$$

Definition of weighted average

$$\approx 56.4$$

Simplify.

Bill drove at an average speed of about 56.4 miles per hour.

Exercises

1. TRAVEL Mr. Anders and Ms. Rich each drove home from a business meeting. Mr. Anders traveled east at 100 kilometers per hour and Ms. Rich traveled west at 80 kilometers per hour. In how many hours were they 100 kilometers apart. **$\frac{5}{9}$ h**

2. AIRPLANES An airplane flies 750 miles due west in $1\frac{1}{2}$ hours and 750 miles due south in 2 hours. What is the average speed of the airplane? **about 429 mph**

3. TRACK Sprinter A runs 100 meters in 15 seconds, while sprinter B starts 1.5 seconds later and runs 100 meters in 14 seconds. If each of them runs at a constant rate, who is further in 10 seconds after the start of the race? Explain.

Sprinter A; since sprinter A runs 100 m in 15 s, this sprinter runs at a rate of $\frac{100}{15}$ m/s. In 10 seconds, sprinter A will have run $\frac{100}{15}(10) = 66.7$ m. Sprinter B's rate is $\frac{100}{14}$. In 10 seconds, with the delayed start, sprinter B has run $\frac{100}{14}(10 - 1.5) = 60.7$ m.

4. TRAINS An express train travels 90 kilometers per hour from Smallville to Megatown. A local train takes 2.5 hours longer to travel the same distance at 50 kilometers per hour. How far apart are Smallville and Megatown? **281.25 km**

5. CYCLING Two cyclists begin traveling in the same direction on the same bike path. One travels at 15 miles per hour, and the other travels at 12 miles per hour. When will the cyclists be 10 miles apart? **$3\frac{1}{3}$ h**

6. TRAINS Two trains leave Chicago, one traveling east at 30 miles per hour and one traveling west at 40 miles per hour. When will the trains be 210 miles apart? **3 h**

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3-9 Study Guide and Intervention

Weighted Averages

Mixture Problems

The weighted average M of a set of data is the sum of the product of each number in the set and its weight divided by the sum of all the weights.

Mixture Problems are problems where two or more parts are combined into a whole. They involve weighted averages. In a mixture problem, the weight is usually a price or a percent of something.

Example Delectable Cookie Company sells chocolate chip cookies for \$6.95 per pound and white chocolate cookies for \$5.95 per pound. How many pounds of chocolate chip cookies should be mixed with 4 pounds of white chocolate cookies to obtain a mixture that sells for \$6.75 per pound.

Let w = the number of pounds of chocolate chip cookies

	Number of Pounds	Price per Pound	Total Price
Chocolate Chip	w	6.95	$6.95w$
White Chocolate	4	5.95	$4(5.95)$
Mixture	$w + 4$	6.75	$6.75(w + 4)$

Equation: $6.95w + 4(5.95) = 6.75(w + 4)$

Solve the equation.

$$6.95w + 4(5.95) = 6.75(w + 4)$$

Original equation

$$6.95w + 23.80 = 6.75w + 27$$

Simplify.

$$6.95w + 23.80 - 6.75w = 6.75w + 27 - 6.75w$$

Subtract $6.75w$ from each side.

$$0.2w + 23.80 = 27$$

Simplify.

$$0.2w + 23.80 - 23.80 = 27 - 23.80$$

Subtract 23.80 from each side.

$$0.2w = 3.2$$

Simplify.

$$w = 16$$

16 pounds of chocolate chip cookies should be mixed with 4 pounds of white chocolate cookies.

Exercises

1. SOLUTIONS How many grams of sugar must be added to 60 grams of a solution that is 32% sugar to obtain a solution that is 50% sugar? **21.6 g**

2. NUTS The Quik Mart has two kinds of nuts. Pecans sell for \$1.55 per pound and walnuts sell for \$1.95 per pound. How many pounds of walnuts must be added to 15 pounds of pecans to make a mixture that sells for \$1.75 per pound? **15 lb**

3. INVESTMENTS Alice Gleason invested a portion of \$32,000 at 9% interest and the balance at 11% interest. How much did she invest at each rate if her total income from both investments was \$3,200. **\$16,000 at 9% and \$16,000 at 11%**

4. MILK Whole milk is 4% butterfat. How much skim milk with 0% butterfat should be added to 32 ounces of whole milk to obtain a mixture that is 2.5% butterfat? **19.2 oz**

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SEASONING For Exercises 1–4, use the following information.

A health food store sells seasoning blends in bulk. One blend contains 20% basil. Sheila wants to add pure basil to some 20% blend to make 16 ounces of her own 30% blend. Let b represent the amount of basil Sheila should add to the 20% blend.

1. Complete the table representing the problem.

	Ounces	Amount of Basil
20% Basil Blend	$16 - b$	$0.20(16 - b)$
100% Basil	b	$1.00b$
30% Basil Blend	16	$0.30(16)$

2. Write an equation to represent the problem. $0.20(16 - b) + 1.00b = 0.30(16)$
3. How many ounces of basil should Sheila use to make the 30% blend? **2 oz**
4. How many ounces of the 20% blend should she use? **14 oz**

HIKING For Exercises 5–7, use the following information.

At 7:00 A.M., two groups of hikers begin 21 miles apart and head toward each other. The first group, hiking at an average rate of 1.5 miles per hour, carries tents, sleeping bags, and cooking equipment. The second group, hiking at an average rate of 2 miles per hour, carries food and water. Let t represent the hiking time.

5. Copy and complete the table representing the problem.

	r	t	$d = rt$
First group of hikers	1.5	t	$1.5t$
Second group of hikers	2	t	$2t$

6. Write an equation using t that describes the distances traveled. $1.5t + 2t = 21$
7. How long will it be until the two groups of hikers meet? **6 h**

SALES For Exercises 8 and 9, use the following information.

Sergio sells a mixture of Virginia peanuts and Spanish peanuts for \$3.40 per pound. To make the mixture, he uses Virginia peanuts that cost \$3.50 per pound and Spanish peanuts that cost \$3.00 per pound. He mixes 10 pounds at a time.

8. How many pounds of Virginia peanuts does Sergio use? **8 lb**
9. How many pounds of Spanish peanuts does Sergio use? **2 lb**

GRASS SEED For Exercises 1–4, use the following information.

A nursery sells Kentucky Blue Grass seed for \$5.75 per pound and Tall Fescue seed for \$4.50 per pound. The nursery sells a mixture of the two kinds of seed for \$5.25 per pound. Let k represent the amount of Kentucky Blue Grass seed the nursery uses in 5 pounds of the mixture.

1. Complete the table representing the problem.

	Number of Pounds	Price per Pound	Cost
Kentucky Blue Grass	k	\$5.75	$5.75k$
Tall Fescue	$5 - k$	\$4.50	$4.50(5 - k)$
Mixture	5	\$5.25	$5.25(5)$

2. Write an equation to represent the problem. $5.75k + 4.50(5 - k) = 5.25(5)$
3. How much Kentucky Blue Grass does the nursery use in 5 pounds of the mixture? **3 lb**
4. How much Tall Fescue does the nursery use in 5 pounds of the mixture? **2 lb**

TRAVEL For Exercises 5–7, use the following information.

Two commuter trains carry passengers between two cities, one traveling east, and the other west, on different tracks. Their respective stations are 150 miles apart. Both trains leave at the same time, one traveling at an average speed of 55 miles per hour and the other at an average speed of 65 miles per hour. Let t represent the time until the trains pass each other.

5. Copy and complete the table representing the problem.

	r	t	$d = rt$
First Train	55	t	$55t$
Second Train	65	t	$65t$

6. Write an equation using t that describes the distances traveled. $55t + 65t = 150$
7. How long after departing will the trains pass each other? **1.25 h**
8. **TRAVEL** Two trains leave Raleigh at the same time, one traveling north, and the other south. The first train travels at 50 miles per hour and the second at 60 miles per hour. In how many hours will the trains be 275 miles apart? **2.5 h**
9. **JUICE** A pineapple drink contains 15% pineapple juice. How much pure pineapple juice should be added to 8 quarts of the drink to obtain a mixture containing 50% pineapple juice? **5.6 qt**

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3-9 Reading to Learn Mathematics

Weighted Averages

Pre-Activity How are scores calculated in a figure skating competition?

Read the introduction to Lesson 3-9 at the top of page 171 in your textbook.

Why is the sum of Ilia Kulik's scores divided by 3?

Her first score is counted once and her second score is counted twice.

Reading the Lesson

- Read the definition of *weighted average* on page 171 of your textbook. What is meant by the weight of a number in a set of data?
- Linda's quiz scores in science are 90, 85, 85, 75, 85, and 90. What is the weight of the score 85?
- Suppose Clint drives at 50 miles per hour for 2 hours. Then he drives at 60 miles per hour for 3 hours.
 - Write his speed for each hour of the trip.
 - What is the weight of each of the two speeds?

Speed	50	50	60	60	60
Hour	1	2	3	4	5

Helping You Remember

4. Making a table can be helpful in solving mixture problems. In your own words, explain how you use a table to solve mixture problems.

Complete each row to write an expression in the last column for each part of the problem and for the combination, then write an equation using those expressions from the last column.

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3-9 Enrichment

Diophantine Equations

The first great algebraist, Diophantus of Alexandria (about A.D. 300), devoted much of his work to the solving of indeterminate equations. An indeterminate equation has more than one variable and an unlimited number of solutions. An example is $x + 2y = 4$.

When the coefficients of an indeterminate equation are integers and you are asked to find solutions that must be integers, the equation is called *diophantine*. Such equations can be quite difficult to solve, often involving trial and error—and some luck!

Solve each diophantine equation by finding at least one pair of positive integers that makes the equation true. Some hints are given to help you.

- $2x + 5y = 32$
 - First solve the equation for x . **$x = 16 - \frac{5y}{2}$**
 - Why must y be an even number? **If y is odd, then x won't be an integer.**
 - Find at least one solution. **Any of these: (11, 2), (6, 4), (1, 6)**
- $5x + 2y = 42$
 - First solve the equation for x . **$x = \frac{42 - 2y}{5}$**
 - Rewrite your answer in the form $x = 8 + \frac{2 - 2y}{5}$
 - Why must $(2 - 2y)$ be a multiple of 5? **Otherwise, x won't be an integer.**
 - Find at least one solution. **Any of these: (8, 1), (6, 6), (4, 11), (2, 16)**
- $2x + 7y = 29$
 - (11, 1) or (4, 3)**
- $7x + 5y = 118$
 - Any of these: (14, 4), (9, 11), (4, 18)**
- $8x - 13y = 100$
 - (19, 4), (32, 12) or any pair when $y = 4n$ and n is a positive odd number**
- $5x - 14y = 11$
 - (5, 1), (19, 6) or any pair when $y = 5m - 4$ and m is a positive number**

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